MHD Free Convective Heat Transfer on Reacting Flow over a Vertical Plate with Constant Thermal Conductivity

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Abstract
This paper considers the effect of constant thermal conductivity on MHD free convective heat transfer fluid flow over a vertical plate. The steady two-dimensional laminar viscous nonlinear partial differential governing equations is transformed into a nonlinear ordinary differential equation using similarity transforms. The resulting problem is solved numerically using Runge-Kutta fourth order technique with shooting method. The effect of constant thermal conductivity on the governing parameter is analysed and the results obtained is displayed graphically and the rate of heat transfer and skin friction is shown in tables.

Keywords: Reacting flow; vertical plate; constant thermal conductivity; MHD; shooting method.

1. Introduction

The problem of free convection has attracted many researchers in view of its application in geophysics, astrophysics, geological formations, and thermal recovery of oil, and in assessment of aquifers, geothermal reservoirs and underground nuclear waste storage site, etc. Also, the growing needs in industries and engineering sectors, requires the study of heat and mass transfer in the presence of different conditions and parameters with reaction flow effects. Basant (1998) considered the effects of applied magnetic field on transient free convective flow in a vertical channel. Acharya et al. (2000) studied magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Mansuor et al. (2008), considered a steady two dimensional nonlinear MHD boundary layer flow of an incompressible, viscous and electrically conducting fluid in the presence of a uniform magnetic field with heat, mass transfer and chemical reaction in a porous medium. Kishore et al. (2010) investigated the unsteady free convection flow of an incompressible viscous fluid past an exponentially accelerated vertical plate, by taking into account the heat due to viscous dissipation under the influence of a uniform transverse magnetic field. Kabir et al. (2013) investigated the influences of viscous dissipation on MHD natural convection flow along a uniformly heated vertical wavy surface.

Magneto convection plays an important role in various industrial applications such as magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors, salt water, collision less plasmas and magnetic suppression of molten semi-conducting materials. Hayat et al. (2014) studied the flow of variable thermal conductivity fluid due to inclined stretching cylinder with viscous dissipation and thermal radiation. Kaprawi (2015) studied analysis of transient natural convection flow past an accelerated infinite vertical plate and the result showed that the temperature and velocity profiles are significantly influenced by Prandtl number and Grashof number. This study extends the work of Oyem et al. (2015) to consider a steady two-dimensional laminar MHD free convective heat transfer reacting flow over a vertical plate assuming that the flow is subject to constant thermal conductivity and there is heat generated by viscous dissipation.

2. Problem Formulation

Consider a steady two-dimensional laminar free convective heat transfer flow of a viscous, incompressible fluid over a vertical plate in the presence of magnetic field effect and viscous dissipation. The x-axis is taken along the vertical plate in the upward direction and the y-axis is normal to the plate. The fluid is reacting with a uniform magnetic field (β_0) applied normal to the flow field and thermal conductivity (κ) of the flow is assumed to be constant. Under Boussinesq's boundary layer approximation, the steady flow is governed by the nonlinear partial differential mass, momentum and energy equations given as (Oyem et al., 2015):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma\beta_0^2 u}{\rho} \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)+\frac{AQexp^{-\frac{E}{RT}}}{\rho c_p}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (3)$$

where u, v are the velocity components in x, y directions respectively, ρ is density of the fluid, c_p is the specific heat capacity at constant pressure, v is the kinematic viscosity, g is the acceleration due to gravity, σ is electrical conductivity, T is temperature of the fluid, β and β_0 are coefficient of volumetric expansion and magnetic field intensity, A is the pre-exponential (frequency) factor, Q is heat release, E is the activation energy, R is the universal gas constant and μ is the fluid viscosity coefficient. The boundary conditions for the velocity and temperature fields is

$$\begin{gathered} u=0 \quad v=0 \quad T=T_w \quad at \quad y=0 \\ u\to 0 \quad T\to T_\infty \quad as \quad y\to\infty \end{gathered} \quad (4)$$

where T_w is the wall dimensional temperature and T_∞ is free stream dimensional temperature. Expanding equation (3) and introducing the stream function $\psi(x,y)$ where

$$u=\frac{\partial\psi}{\partial y} \quad \text{and} \quad v=-\frac{\partial\psi}{\partial x} \quad (5)$$

From (5), equation (1) is satisfied and equations (2) and (3) with boundary conditions (4) are transformed. Introducing the following similarity and dimensionless variables

$$\eta=y\sqrt{\frac{U_0}{2vx}} \quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \quad \psi=\sqrt{2xvU_0}\,f(\eta) \quad u=U_0 f'(\eta) \quad (6)$$

one obtains the coupled nonlinear differential equation subject to the boundary conditions given as;

$$\frac{\partial^3 f}{\partial\eta^3}+f\frac{\partial^2 f}{\partial\eta^2}+Gr\theta-M\frac{\partial f}{\partial\eta}=0 \quad (7)$$

$$\frac{\partial^2\theta}{\partial\eta^2}+Prf\frac{\partial\theta}{\partial\eta}+\delta exp^{\left(\frac{\theta}{\varepsilon\theta+1}\right)}+PrEc\left(\frac{\partial^2 f}{\partial\eta^2}\right)^2=0 \quad (8)$$

$$\left.\begin{array}{lllll} f=0, & f'=0, & \theta=1 & at & \eta=0 \\ f'=0, & \theta=0 & & as & \eta\to\infty \end{array}\right\} \quad (9)$$

where f is the dimensionless velocity of the fluid, $Gr=\frac{2xg\beta(T_w-T_\infty)}{U_0^2}$ is the local Grashof number and $M=\frac{2x\sigma\beta_0^2}{\rho U_0}$ is the local magnetic field parameter of the flow, θ is the dimensionless temperature, $Pr=\frac{\upsilon}{\alpha}$ is the Prandtl number, $\varepsilon=\frac{RT\infty}{E}$ is activation energy parameter, $\delta=\frac{\upsilon}{\alpha}\frac{2xAQexp^{-\frac{E}{RT\infty}}}{\rho C_p U_0(T_w-T_\infty)}$ is modified Frank-Kamenetskii parameter and $Ec=\frac{U_0^2}{C_p(T_w-T_\infty)}$ is the Eckert number. The principal physical quantities are the wall shear stress τ_w in terms of skin-friction coefficient C_f and rate of heat transfer in terms of the Nusselt number Nu defined by;

$$C_f=2\left(\frac{\upsilon}{2xU_0}\right)^{\frac{1}{2}}\left(\frac{\partial^2 f}{\partial\eta^2}\right)_{y=0} \text{ and } Nu=\left(\frac{2x\upsilon}{U_0}\right)^{\frac{1}{2}}\frac{q_w}{\kappa(T_w-T_\infty)}=-\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}$$

were $\tau_w=\mu\left.\frac{\partial u}{\partial y}\right|_{y=0}$ and $q_w=-\kappa(T)\left.\frac{\partial T}{\partial y}\right|_{y=0}$.

3. Numerical Computation

In order to solve the steady, nonlinear coupled ordinary differential equations (7) and (8) with the initial and boundary conditions (9), shooting method with Runge-Kutta fourth order technique is employed. First of all, higher order nonlinear differential equations (7) and (8) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting method. The resultant initial value problem is solved by employing Runge-Kutta fourth order technique with step-size $\Delta\eta=0.001$ to obtain the numerical solution with four decimal place accuracy as the criterion of convergence. From the process of numerical computation the skin friction coefficient and the Nusselt number are also obtained and presented in a tabular form.

4. Results and Discussions

In order to develop the physical insight of the governing boundary layer problem, parameters of the flow Gr, M, Pr, δ, ε and Ec are taken from Omowaye and Koriko (2014) and at constant thermal conductivity, the effects of the local Grashof number Gr, local magnetic field parameter M, Prandtl number Pr, modified Frank-Kamenetskii parameter δ, activation energy ε and Eckert number Ec on velocity and temperature profiles are displayed in figures 1 – 12. The effect of Prandtl number Pr on velocity is illustrated in figure 1. It is observed that the velocity decreases as the Prandtl number increases (figure 1). From the figure, velocity boundary layer decreases slightly along the vertical plate towards the free stream thereby characterizing the ratio of thickness of the viscous and thermal boundary layers. The effect of local magnetic field parameter M on velocity profiles with $Pr=0.071$, $Gr=2$, $Ec=1$, $\varepsilon=0.1$ and $\delta=0.01$ is illustrated in figure 2. It is observed that velocity decreases as the local magnetic field parameter increases. It is because that the application of transverse magnetic field will result in a resistive type force, known as Lorentz force, which tend to resist the fluid flow field and thus reduces velocity.

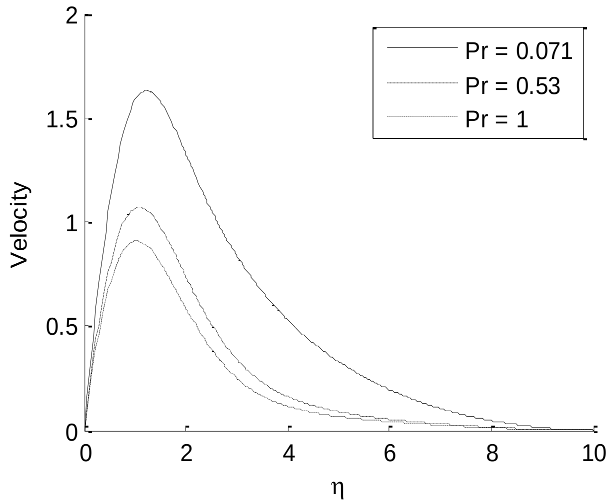

Figure 1: Velocity profiles for different values of Prandtl number Pr

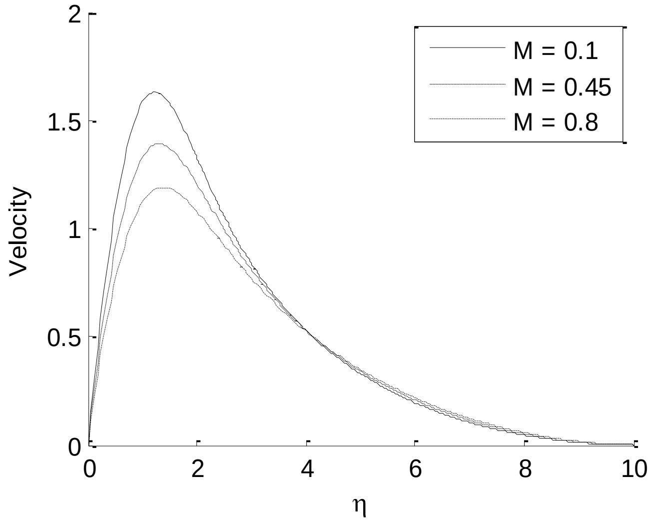

Figure 2: Velocity profiles for different values of local magnetic field

The effect of local Grashof number (Gr) and Eckert number (Ec) for heat and mass transfer on the velocity of the flow filed is presented in figures 3 and 4. In figure 3, the velocity of the flow field is observed to increase with increasing values of local Grashof number. This is due to enhancement of buoyancy force. Similarly, it is observed from figure 4 that increasing values of Ec leads to increase in the velocity distribution in the flow region. This is due to the heat energy stores in the fluid because of frictional heating. Effect of modified Frank-Kamenetskii δ parameter on velocity with $\varepsilon = 0.1$, $M = 0.1$, $Gr = 2$, $Ec = 1$ and $Pr = 0.071$ is presented in figure 5. It is clear that velocity near to the vertical plate increases as δ increases. But the reverse is the case with the effect of activation energy parameter on velocity as shown in figure 6. It is observed that velocity decreases slightly away from the plate towards the free stream.

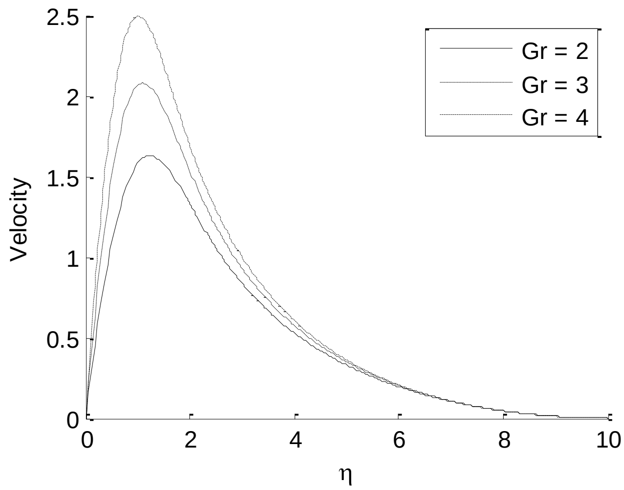

Figure 3: Velocity profiles for different values of Gr

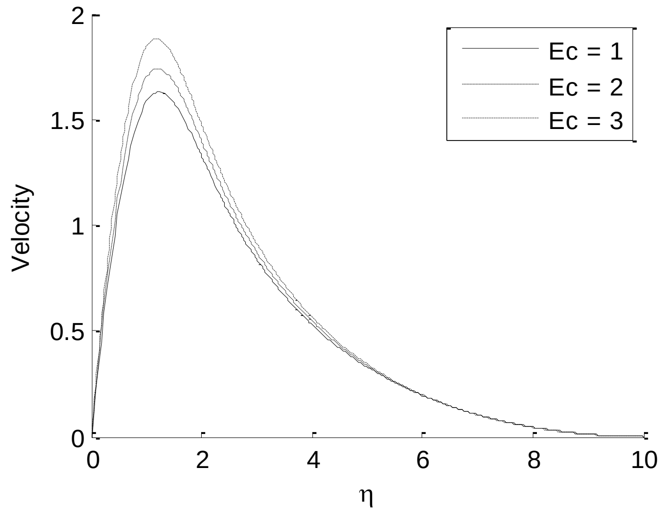

Figure 4: Velocity profiles for different values of Ecker number Ec

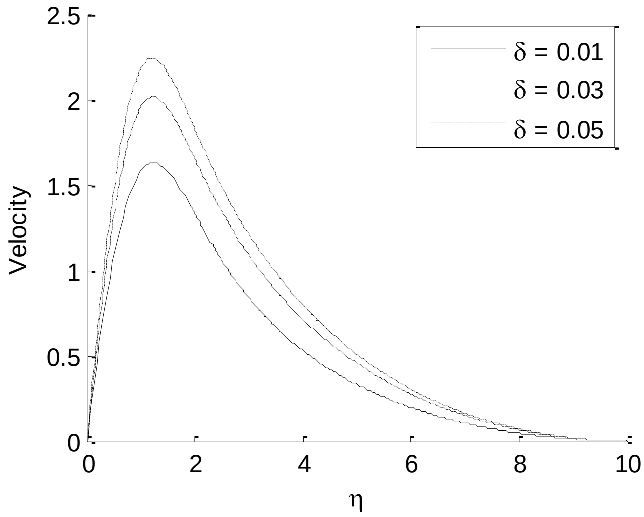

Figure 5: Velocity profiles for different values of δ

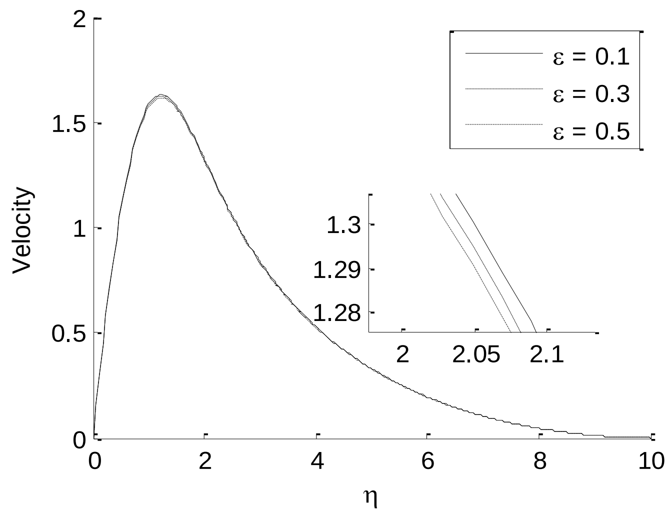

Figure 6: Velocity profiles for different values of activation energy ε

The effects of Prandtl number Pr, local magnetic field parameter M, local Grashof number Gr, Eckert number Ec, activation energy parameter ε and modified Frank-Kamenetskii parameter δ on temperature are shown in figures 7 – 12. The effect of Pr on temperature profile is presented in figure 7. It is observed that the temperature decreases sharply with increasing values of Pr in the vicinity of the plate. The effect of temperature for different values of M with prescribed values is presented in figure 8. It is observed that temperature decreases with increasing values of M initially within the vicinity of the plate and then increases in the remaining flow region. The effect of different values of local Grashof number on temperature distribution is illustrated in figure 9 and temperature profiles for different values of Ec is shown in figure 10.

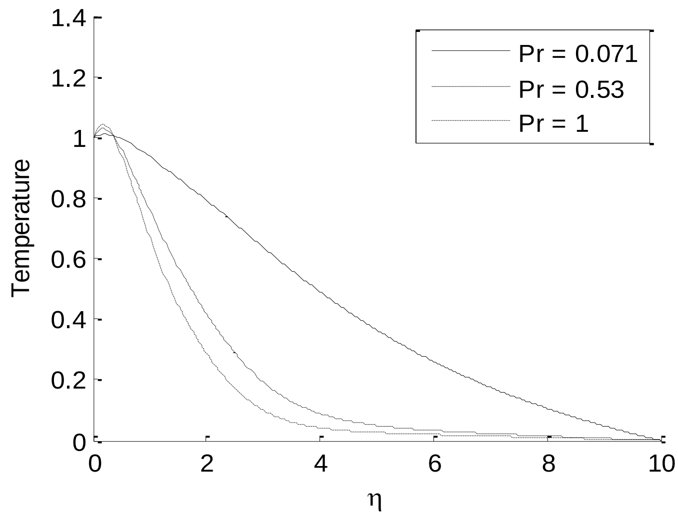

Figure 7: Temperature profiles for different values of Prandtl number

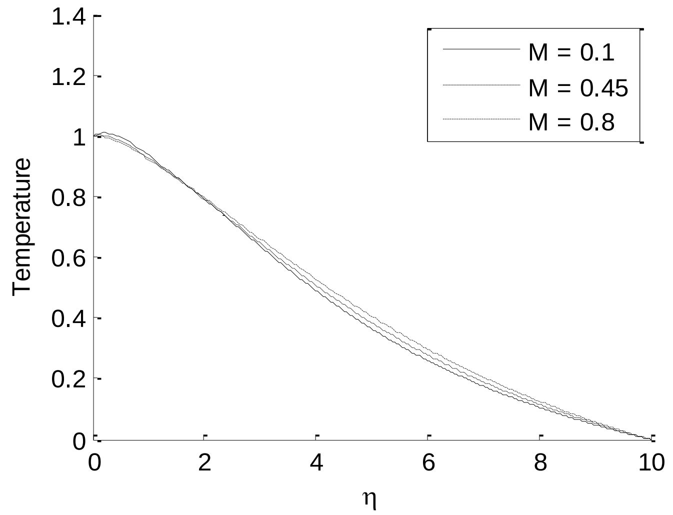

Figure 8: Temperature profiles for different values of M

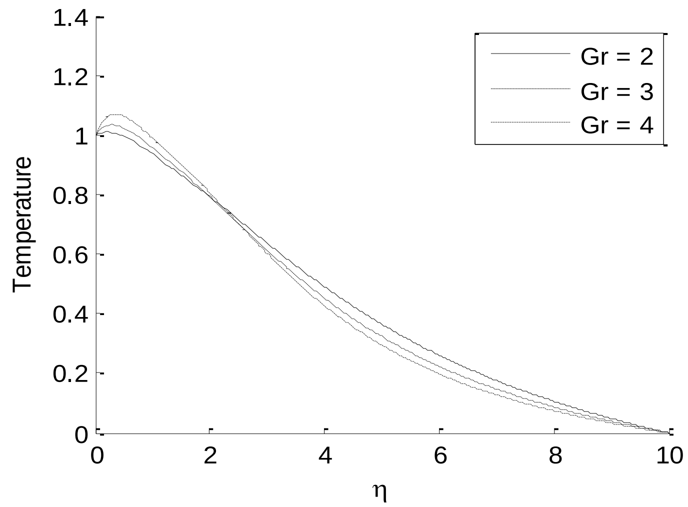

Figure 9: Temperature profiles for different values of Gr

It is observed from figure 9 that temperature near the vertical plate increases as Gr increases but, an opposite effect is noticed at a certain distance from the plate ($\eta_0 \cong 2.5$), thereby decreasing away from the plate towards the free stream velocity. It is also observed from figure 10 that temperature increases with increase in Eckert number. As Ec increases in value, temperature increases sharply from the plate and gradually towards the free stream in the flow region. This is due to the fact that heat energy is stored in liquid due to frictional heating. Thus, the effect of increasing Eckert number is to enhance the temperature at any point as well as the velocity. The effect of activation energy parameter on temperature profiles is presented in figure 11. It is clear that temperature profiles decreases slightly with increasing values of activation energy parameter ε away from the plate.

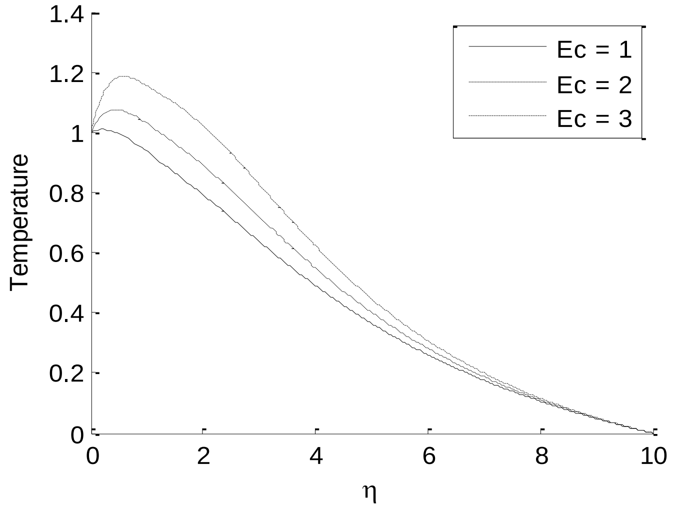

Figure 10: Temperature profiles for different values of Ec

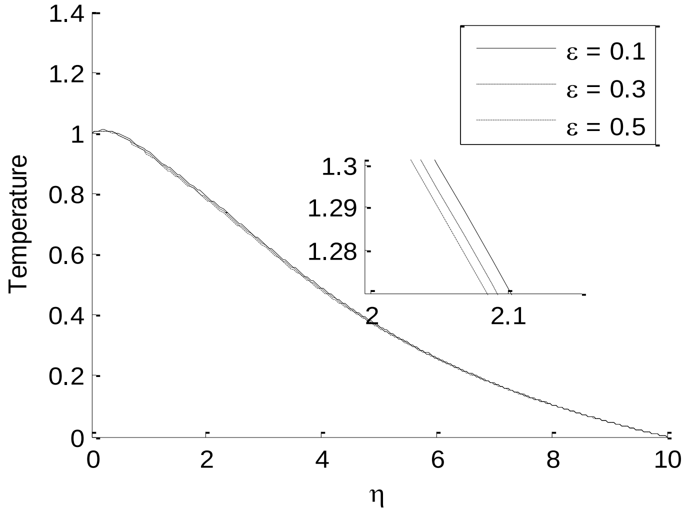

Figure 11: Temperature profiles for various values of ε

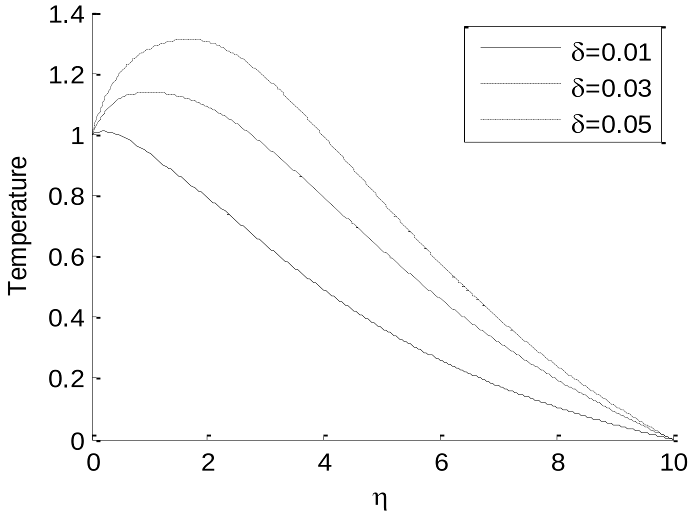

Figure 12: Temperature profiles for various values of δ

The effect of temperature on different values of modified Frank-Kamenetskii parameter δ in presented in figure 12. It is observed that as δ increases, temperature increases also. Numerical values of skin friction coefficient and rate of heat transfer are presented in table 1. It is observed that an increase in Pr, M and ε, leads to a decrease in skin friction coefficient and decrease in heat transfer rate with increasing values of M and ε. Meanwhile, increase in Pr, Gr Ec and δ leads to increase in Nusselt number while increase in skin friction coefficient results in the increase of Gr, Ec and δ.

Table 1: Numerical values of skin friction coefficient and Nusselt number

Pr	M	Gr	Ec	ε	δ	C_f	Nu
0.071	0.1	2	1	0.1	0.01	2.6767	0.0942
0.53	0.1	2	1	0.1	0.01	2.0816	0.3383
1	0.1	2	1	0.1	0.01	1.9049	0.5142
0.071	0.45	2	1	0.1	0.01	2.2760	0.0403
0.071	0.8	2	1	0.1	0.01	1.9675	0.0076
0.071	0.1	3	1	0.1	0.01	3.7759	0.2450
0.071	0.1	4	1	0.1	0.01	4.8691	0.4425
0.071	0.1	2	2	0.1	0.01	2.8634	0.3812
0.071	0.1	2	3	0.1	0.01	3.1069	0.7751
0.071	0.1	2	1	0.3	0.01	2.6656	0.0860
0.071	0.1	2	1	0.5	0.01	2.6576	0.0803
0.071	0.1	2	1	0.1	0.03	3.1899	0.3957
0.071	0.1	2	1	0.1	0.04	3.5175	0.6086

5. Conclusion

An analysis is carried out for free convective flow and heat transfer of a reacting flow over a vertical plate in the presence of constant thermal conductivity effects. The present study indicates that due to increase in viscous dissipation in terms of Eckert number (Ec), velocity and temperature increases. While increase in magnetic field parameter (M) results in a decrease in both velocity and temperature of the fluid flow as it helps in controlling the flow. The skin friction coefficient decreases with increasing values of Prandtl number (Pr) and magnetic field parameter (M) but increases when viscous dissipation (Ec), activation energy parameter (ε), Grashof number (Gr) and modified Frank-Kamenetskii parameter (δ) increase. Similarly, decrease in Nusselt number results in Prandtl number and increase in magnetic field parameter.

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